3D Conformal Transformation

http://www.fc.up.pt/pessoas/jagoncal/coord/3DCONF.html

The absolute orientation process of a photogrammetric stereo pair is made by a 3D conformal transformation. The relative orientation process generates 3D coordinate of conjugate points in an arbitrary reference system (u, v, w). Knowing the position of at least 3 non collinear points in this system and in some other reference system, (x, y, z), the transformation is given by:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = S \cdot R_3(\kappa) \cdot R_2(\varphi) \cdot R_1(\omega) \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix}$$

where S is a scale factor, $(\omega, \varphi, \kappa)$, are rotation angles around axis (u, v, w), respectively, and (T_x, T_y, T_z) are translation parameters. The formula can be expressed also as:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = S \cdot \begin{pmatrix} \cos\kappa & sin\,\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = S \cdot \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix}$$

where

$$\begin{aligned}
r_{11} &= \cos\varphi\cos\kappa \\
r_{12} &= \sin\omega\sin\varphi\cos\kappa + \cos\omega\sin\kappa \\
r_{13} &= -\sin\omega\sin\varphi\cos\kappa + \cos\omega\sin\kappa \\
r_{21} &= -\cos\varphi\sin\kappa \\
r_{22} &= -\sin\omega\sin\varphi\sin\kappa + \sin\omega\sin\kappa \\
r_{23} &= \cos\omega\sin\varphi\sin\kappa + \sin\omega\cos\kappa \\
r_{31} &= \cos\varphi\cos\kappa \\
r_{32} &= -\sin\omega\cos\varphi \\
r_{13} &= \cos\omega\cos\varphi
\end{aligned}$$

The program takes a set of 3 or more control points, with known coordinates in both systems, and determines the 7 parameters by least squares adjustment. The (x, y, z) system can be a map projection but it should ideally be a local geodetic system.

The program provides the resulting parameters, the residuals and the Root Mean Square (RMS) errors. It also can take a set of points in (u, v, w) coordinates and apply the formula to calculate the corresponding (x, y, z) coordinates.

Initial approximations are obtained according to Dewitt (1996). The program was adapted by a program in C language developed by David Haydock, PhD student of University College London, Department of Photogrammetry and Surveying, in 1991.

Reference:

Dewitt, B. A. (1996). Initial approximations for the three-dimensional conformal coordinate transformation. *Photogrammetric engineering and remote sensing*, 62(1), 79-84.

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